1 Introduction to quantum mechanics

Bring solutions along to the class at 10am on 5/10/20.

Questions marked # are beyond the syllabus. We may discuss them in the class if there is time. Attempt all questions. Feel free to discuss with friends. Don't worry if you cannot answer a question fully as you will discuss the solutions in the class.

1.0 Videos

Please watch this week's videos: V1.0, V1.1, V1.2, V1.3, V1.4, V1.5.
All videos are available on the youtube channel Introductory Quantum Mechanics (link available at felixflicker.com/teaching, or on Learning Central).

1.1 Probability amplitudes

List reasons why $\psi(x,t)$ is not suitable to interpret as a probability density, but $|\psi(x,t)|^2$ is.

1.2 Quantum scales

A simple estimate as to whether quantum behaviour needs to be accounted for when modelling a physical system is provided by estimating a relevant quantity with the same units as $\hbar$. For example, an energy scale multiplied by a time scale, or a momentum scale multiplied by a length scale. If the quantity is significantly larger than $\hbar$ probably quantum effects can be neglected, otherwise they may be important. Estimate whether quantum effects are likely to be important in the following cases. You should be happy defending your answers to your classmates.
(a) A game of cricket
(b) the behaviour of electrons in a transistor
(c) bacteria swimming
(d) neurons.

1.3 The Schrödinger equation

The time dependent Schrödinger equation (TDSE) is

$$i\hbar\partial_t\psi(x,t) = \hat{H}\psi(x,t)\,. \tag{37}$$

1.3.1

By defining

$$\psi(x,t) = \phi(x)\,T(t) \tag{38}$$

show that the TDSE is separable provided the Hamiltonian $\hat{H}$ has no explicit time dependence. Derive the time independent Schrödinger equation (TISE):

$$\hat{H}\phi(x) = E\phi(x) \tag{39}$$

and solve the corresponding equation for $T(t)$.

1.3.2

For a solution to the TISE state the corresponding solution to the TDSE.

1.3.3

A plane wave takes the form

$$\psi_L(x,t) = A_L \exp(i(-kx - \omega t)) \tag{40}$$

$$\psi_R(x,t) = A_R \exp(i(kx - \omega t)) \tag{41}$$

where L and R indicate leftgoing and rightgoing waves.

(a) Show that the Schrödinger equation returns the Einstein relation $E = \hbar\omega$.

(b) Show that the Schrödinger equation returns the de Broglie relation $p = \hbar k = h/\lambda$.

1.3.4

If $\psi(x,t)$ describes a state evolving forwards in time, its complex conjugate $\psi(x,t)^*$ can be thought of as describing a state evolving backwards in time. Assuming a purely real Hamiltonian, explain using the TDSE why this interpretation is natural.

1.4 Probability current density

1.4.1

Defining the probability density $\rho(x,t) = |\psi(x,t)|^2$ show that

$$\partial_t \rho + \partial_x j = 0 \tag{42}$$

where

$$j(x,t) = -\frac{i\hbar}{2m}\left(\psi^* \partial_x \psi - \psi \partial_x \psi^*\right). \tag{43}$$

Interpret the result physically.

1.4.2

Find $j(x,t)$ for the plane waves in 1.2.3, and show that j describes the velocity of the waves.