# Double Integrals

In the spirit of last week's line integral 5-point plan, here's a 5-point plan for doing double integrals.

1. Draw all four boundary lines.
2. Mark the area to be integrated. It can be helpful to shade the region in with straight lines perpendicular to the direction of the outer integral.
3. Evaluate the inner indefinite integral.
4. Substitute the limits.
5. You should now be left with a standard 1D integral. Evaulate it.

It can be helpful to write the limits on the integrals in the form

$$\int_{x=a}^{x=b} (stuff)\,\mathrm{d}x$$

where you might normally just write

$$\int_{a}^{b} (stuff)\,\mathrm{d}x.$$

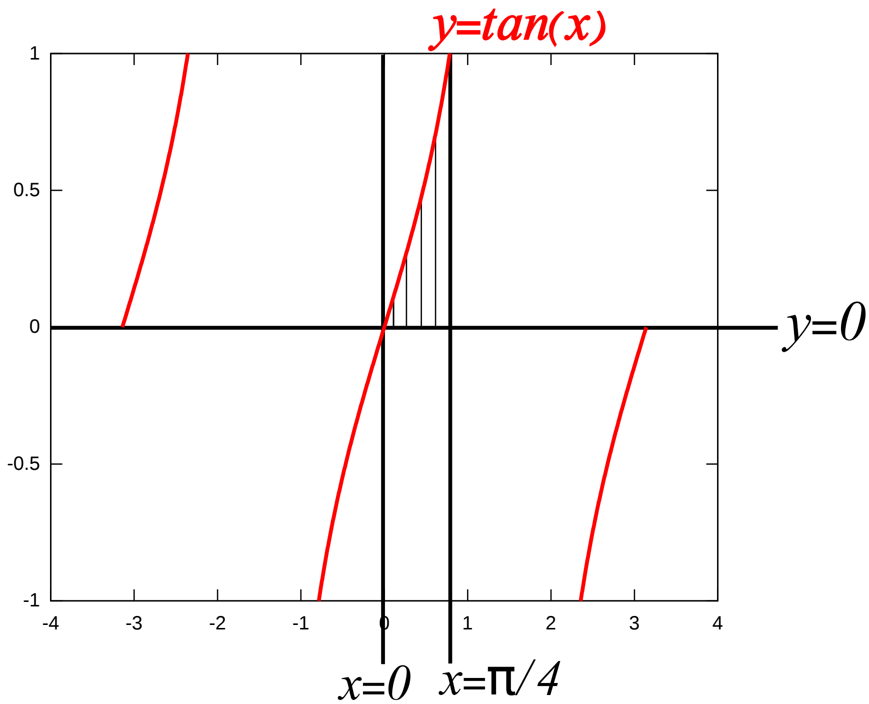
That way the boundary lines used in (1) are clear. Even if the question doesn't ask for a picture you should draw one anyway for this kind of problem.
Let's take as an example PS6 Q1b:

$$\int_{0}^{\pi/4}\int_{0}^{\tan(x)} \sec(x)\,\mathrm{d}y\mathrm{d}x$$

first, write the limits more explicitly, and add some parentheses to make things clear:

$$\int_{x=0}^{x=\pi/4}\left(\int_{y=0}^{y=\tan(x)} \sec(x)\,\mathrm{d}y\right)\mathrm{d}x.$$

Points (1) and (2) are shown below. Note that the outer integral is a $\mathrm{d}x$, so the stripes are separated by small shifts $\mathrm{d}x$ along the $x$-axis.




(3)

$$\int_{y=0}^{y=\tan(x)} \sec(x)\,\mathrm{d}y = [y\sec(x)]_{y=0}^{y=\tan(x)}$$

(4)

$$[y\sec(x)]_{y=0}^{y=\tan(x)} = \tan(x)\sec(x)$$

(5) the remaining integral is therefore

$$\int_{x=0}^{x=\pi/4} \tan(x)\sec(x)\,\mathrm{d}x = \int_{x=0}^{x=\pi/4} \frac{\sin(x)}{\cos^2(x)}\mathrm{d}x$$

and in this case we see by inspection that the result is

$$\left[\frac{1}{\cos(x)}\right]_{x=0}^{x=\pi/4} = \sqrt{2}-1.$$