PX2132: Introductory Quantum Mechanics
Syllabus

September 20, 2021

Module information

- Autumn semester: 10 credits.
- Module Organiser: Dr F Flicker.
- Deputy Module Organiser: Dr S Ladak.
- Teaching and feedback methods:
  - pre-recorded online material totalling 11hrs
  - weekly reading to be completed before the lecture
  - live discussion classes, 22×1hr
- Assessment:
  - Examination 60%: part A multiple choice, part B written answers [Examination duration: 2 hours]
  - Weekly problem sets: 1% for satisfactory completion of each of 10 problem sets, defined as a meaningful attempt at every question
  - Coursework 1: 10%
  - Coursework 2, group poster: 5% (peer assessed)
  - Coursework 2, presentation: 5% (peer assessed)
  - Coursework 2, contribution to team project: 10% (peer assessed)
- Re-assessment : 100% multiple choice examination.
- Pre-cursors: PX1120*, PX1221 and PX1230* (*excepting joint Maths/Physics students).
- Co-requisites: PX2131.
- Pre-requisites: None.

Aims of the module

- To provide foundations of the description of matter by wave mechanics, in particular through the Schrödinger equation and the interpretation and use of the wave function.
- To introduce more formal aspects of wave mechanics.
- To use worked examples and model systems to develop understanding of the meaning of wave functions, eigenvalues, eigenfunctions and operators.
- To apply quantum mechanics to describing the hydrogen atom.

Learning outcomes

The student will be able to:

- recall and use basic quantum-mechanical concepts, including Schrödinger's time-independent and time-dependent wave equations, expectation values, operators, and the uncertainty principle
- find normalised energy eigenfunctions and eigenvalues in some simple potentials
- describe scattering from step potentials
- describe quantum-mechanical tunnelling
- mathematically describe the time evolution of quantum states
- understand and use Dirac notation
- describe the mathematical structure of spin-1/2 systems
- appreciate how angular momentum appears in quantum mechanics. Solve problems relating to this and quantum states of the hydrogen atom.

Books

The course will be based around the following books which are freely available online:

- J. Binney and D. Skinner, *The Physics of Quantum Mechanics* [https://www-thphys.physics.ox.ac.uk/people/JamesBinney/QBhome.htm]
- P. A. M. Dirac, *The Principles of Quantum Mechanics* [archive.org/details/in.ernet.dli.2015.177580]
- R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* [feynmanlectures.caltech.edu]

While later editions of the first two books are available for a price, references should be assumed to be made to these free editions. The following books are available as eBooks for free through Cardiff University:

- D. J. Griffiths, *Introduction to Quantum Mechanics* (Cambridge University Press, 2nd edition)
- S. Weinberg, *Lectures on Quantum Mechanics* (Cambridge University Press, 2nd edition, 2015).

Syllabus

1 Introduction

Videos:

- V1.0: Introduction to the course
- V1.1: History of quantum mechanics
- V1.2: The Schrödinger equation
- V1.3: Plane waves
- V1.4: Amplitudes and probabilities
- V1.5: Two slit demo

Topics:

- recap of vectors, matrices, and differential equations
- the experimental necessity of quantum mechanics: Compton scattering, de Broglie relation $p = h/\lambda = \hbar k$; single particle interference; photoelectric effect and $E = hf = \hbar\omega$
- the time-dependent Schrodinger equation (TDSE)
- the time-independent Schrodinger equation (TISE)
- the wavefunction
- probability density
- probability current density
- general boundary conditions.

For the exam you should be able to:

- understand and apply $p = \hbar k$, $E = \hbar\omega$
- write down the TDSE and TISE
- derive the TISE from the TDSE using separation of variables
- deduce the time dependence of a solution to the TISE
- state the Born rule
- state the meaning of the probability density and to calculate it for a given wavefunction
- derive the continuity equation for the local conservation of probability
- derive the probability current density and calculate it for a given wavefunction
- state the two boundary conditions which always apply to the wavefunction.

2 Scattering and tunnelling

Videos:

- V2.1a–V2.1d: Scattering from a potential step
- V2.2: Quantum tunnelling
- V3.3: Evanescent waves demo

Topics:

- plane waves
- recovering $p = \hbar k$ and $E = \hbar\omega$ from the Schrödinger equation
- scattering from a potential step
- tunnelling and barrier penetration
- scanning tunnelling microscopes.

For the exam you should be able to:

- write down the form of a plane wave
- use this form to show the Schrödinger equation is compatible with $p = \hbar k$ and $E = \hbar\omega$

- state the forms of the TISE in regions of constant potential
- find the transmission and reflection amplitudes for scattering from a potential step
- find the probability current densities for scattering from a potential step
- find the probabilities of transmission and reflection from a potential step
- explain the steps necessary to solve scattering from a potential barrier of finite width
- explain the physical significance of quantum tunnelling
- explain the relevance to scanning tunnelling spectroscopy and microscopy

For the exam you will not be required to:

- rote learn any solutions
- solve explicitly for the amplitudes associated with the finite-width barrier

3 Bound states (I)

Videos:

- V3.1: The infinite potential well
- V3.2: Normalisation
- V3.3: Stationary states
- V3.4: Orthonormality of eigenstates
- V3.5: Fourier decomposition

Topics:

- The infinite potential well (particle in a box)
- energy eigenvalues and eigenfunctions
- the Born rule
- normalisation of wavefunctions
- orthogonality of eigenstates
- energy eigenstates as stationary states
- complete orthonormal bases.

For the exam you should be able to:

- solve for the energy eigenstates and eigenvalues of the infinite potential well (particle in a box)
- explain the physical relevance of normalisation
- normalise a given wavefunction
- explain the relevance of the orthogonality of eigenstates
- demonstrate the orthogonality of given wavefunctions
- explain what is meant by energy eigenstates being stationary states and to prove this mathematically
- explain the significance of sets of eigenstates forming complete orthonormal bases
- explain the significance of expectation values of observable quantities (observables)
- find the expectation values of powers of position and momentum for a given wavefunction

4 Bound states (II)

Videos:

- V4.1: Quantum superposition
- V4.2: The finite potential well

Topics:

- Quantum superposition
- Schrodinger's cat
- the measurement problem
- the finite potential well.

For the exam you should be able to:

- explain the principle of quantum superposition
- calculate properties of superposed states
- decompose a given wavefunction into a superposition of energy eigenstates
- find the time dependence of a given spatial wavefunction
- justify the forms of the wavefunctions solving the TISE for the finite potential well
- explain the steps involved in solving the TISE in the finite potential well
- prove that there is at least one bound state in any finite potential well

For the exam you will not be required to:

- provide a full solution for the finite potential well

5 Finite-dimensional Hilbert spaces

Videos:

- V5.1: Complex vectors
- V5.2: Hermitian matrices
- V5.3a–V5.3c: Spin-1/2
- V5.4: Polarisation demo

Topics:

- Complex vectors and matrices
- Dirac notation
- Hermitian matrices: eigenvalues and eigenvectors, properties
- complete orthonormal bases, resolution of the identity
- Spin-1/2: Stern-Gerlach experiment, Pauli matrices, commutation relations

For the exam you should be able to:

- work with complex vectors and matrices
- employ Dirac notation for complex vectors
- state and derive the properties of Hermitian matrices of use in quantum mechanics
- describe spin-1/2 particles using a 2-dimensional Hilbert space

6 Matrix mechanics (II): the Heisenberg picture

Videos:

- V6.1: Operators and observables
- V6.2: The Heisenberg uncertainty principle
- V6.3: The Heisenberg picture
- V6.4: Conserved quantities

Topics:

- $\hat{p}$ and $\hat{x}$ operators
- canonical commutation relations
- complete sets of states
- quantum numbers
- the Heisenberg uncertainty principle
- the Heisenberg and Schrodinger pictures
- the Heisenberg equations of motion
- Ehrenfest's theorem.

For the exam you should be able to:

- state the canonical commutation relations
- find the commutators of given operators
- explain the significance of operators commuting
- state the Heisenberg uncertainty principle
- show that given wavefunctions obey the uncertainty principle
- explain what is meant by the Heisenberg and Schrodinger pictures
- deduce the Heisenberg equation of motion
- state Ehrenfest's theorem and explain its physical significance
- discuss the correspondence principle
- deduce Ehrenfest's theorem from the Heisenberg equation of motion
- state the meaning of a conserved quantity
- show that the observable quantity associated to a given operator is conserved
- explain the meaning of quantum numbers.

For the exam you will not be required to:

- derive the Heisenberg uncertainty principle.

7 Quantum mechanics

Videos:

- V7.1: Infinite dimensional Hilbert spaces
- V7.2: Fourier transforms
- V7.3: Differential operators
- V7.4: The postulates of quantum mechanics
- V7.5: Schrödinger's cat demo

Topics:

- Functions as innite-dimensional vectors: examples of operators
- Equivalence of Schrödinger, Heisenberg, and Dirac notations
- expectation values
- wavefunction overlap
- The postulates of quantum mechanics
- interpretations of quantum mechanics

For the exam you should be able to:

- work with functions as elements of a vector space, including taking inner products
- state the forms of the operators $\hat{\mathrm{H}}$, $\hat{\mathrm{V}}$, $\hat{\mathrm{p}}$, and $\hat{\mathrm{x}}$ in the position basis
- confirm the Hermiticity of given differential operators in the position basis

For the exam you will not be required to:

- recount details of dierent interpretations of quantum mechanics
- understand the dead cat.

8 The quantum harmonic oscillator

Videos:

- V8.1: The quantum harmonic oscillator
- V8.2: Ladder operators
- V8.3: The number operator
- V8.4: Second quantisation

Topics:

- converting the quantum harmonic oscillator (QHO) TISE to Hermite's equation
- solution with Hermite polynomials
- raising and lowering (ladder) operators
- ladder operator commutation relations
- Energy eigenstates and eigenvalues of the QHO

- Second quantization

For the exam you should be able to:

- work with Hermite polynomials, checking properties such as orthogonality
- find the commutators between the raising and lowering (ladder) operators, and with the Hamiltonian
- demonstrate that these commutation relations lead to an infinite ladder of equally-spaced energy eigenvalues
- justify the normalisation of the ladder operators
- deduce the ground state of the QHO from the existence of a bottom rung of the ladder
- explain the concepts of first and second quantization.

For the exam you will not be required to:

- rote learn the form of the Hermite polynomials
- rote learn the form of the ladder operators.

9 The Schrödinger equation in three dimensions

Videos:

- V9.1: The 3D innite potential well
- V9.2: The 3D quantum harmonic oscillator
- V9.3: Angular momentum
- V9.4: Angular momentum ladder operators

Topics:

- 3D innite potential well
- 3D quantum harmonic oscillator
- polar co-ordinates
- angular momentum
- commutation relations for angular momentum operators
- $\hat{L}_z$ and $\hat{L}^2$ as a maximal set of commuting operators
- angular momentum ladder operators

For the exam you should be able to:

- solve the TISE/TDSE for the 3D infinite potential well
- solve the TISE/TDSE for the 3D quantum harmonic oscillator
- state the form of the angular momentum operator
- derive the position-basis form of the angular momentum operator in cartesian co-ordinates
- derive the position-basis form of $\hat{L}_z$ in polar co-ordinates
- find the commutation relations between the x, y, and z-projections of the angular momentum operator
- show that $\hat{L}^2$ commutes with all three of $\hat{L}_{x,y,z}$
- find the commutation relations between the angular momentum raising and lowering operators $\hat{L}_\pm$
- use the commutation relations between $\hat{L}_\pm$ to deduce the existence of a ladder of states with different $\hat{L}_z$ eigenvalues.

For the exam you will not be required to:

- rote learn the form of ∇^2 in spherical polar co-ordinates.

10 The hydrogen atom

Videos:

- V10.1: Spherically symmetric potentials: angular equation
- V10.2: Spherically symmetric potentials: radial equation
- V10.3: The hydrogen atom

Topics:

- the TISE in spherical polar co-ordinates
- separating into radial and angular equations
- separating the angular equation into the azimuthal and polar equations
- solution of the azimuthal equation using associated Legendre polynomials
- solution of the angular equation using spherical harmonics
- rewriting the radial equation as the 1D TDSE with a centrifugal barrier term
- the TISE for the electron in the hydrogen atom
- solution to the radial equation by reduction to La Guerre's equation
- quantum numbers of the electron in the hydrogen atom

For the exam you should be able to:

- separate the TISE in spherical polar co-ordinates into radial, azimuthal, and polar parts (given the TISE itself)
- rewrite the radial equation of the TISE for a spherically-symmetric potential as a 1D TISE with centrifugal barrier term
- explain the origin of the quantum numbers of the electron in the hydrogen atom
- state the origin of atomic line spectra
- recount the basic idea of the Bohr model of the atom

For the exam you will not be required to:

- rote learn the forms of the spherical harmonics (although you should have a basic familiarity with them)
- learn detailed properties of the associated Legendre equation or La Guerre's equation
- rote learn the solutions to the TISE for the hydrogen atom.