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ESSAYS

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Pattern at the boundaries of order

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Abstract
Chaos, in the sense used by mathematicians and physicists, contains a beautiful co-mingling of order and disorder. Here, we consider the interplay between periodic order, and disorder, in both space and time. This boundary is considered in both mathematical terms, and in terms of our experiences of order and pattern in our everyday routines. Periodic spatial patterns can exist in two dimensions, and finite sections can be made on paper (drawings, designs). These two-dimensional patterns are described by the seventeen 'wallpaper groups', a mathematical classification also used to describe the structure of crystals found in nature. In this article, supported by discussion of disrupted, non-regular and quasiperiodic patterns, we present a series of Lucy's drawings, showing how one periodic order can deform into another. The drawings attempt to represent experienced patterns (habits and routines) as decorative pattern.

Introduction: Periodicity in space and time

Chaos theory is a branch of mathematics devoted to the study of deterministic yet unpredictable systems. Perhaps it is this fundamental embrace of paradox that makes chaos theory so applicable to the real world, having relevance to such disparate fields as animal populations (May 1976), weather systems (Strogatz 1994), chemical reactions (Roux, et al. 1983; Argoul 1987), hydrodynamics (Giglio et al. 1981), and electronics (Linsay 1981; Testa et al. 1982). The name chaos suggests a certain erraticness; yet there is method in the madness, and chaotic systems conceal within them a great deal of 'periodicity', behaviour in which the same situation is returned to at regular intervals. It is this friction between periodicity and disorder that we consider here, with considerations to both space and time.

In space, periodic patterns appear in the structure of crystals. Part of the beauty of crystals, such as diamond, derives from their clear geometric features, with flat faces and regular angles even when growing naturally. This neat geometry persists to the atomic scale; the atoms are arranged into perfectly regular patterns, described with the mathematics of space groups (of which there are 230 in three dimensions). Two-dimensional crystals also exist, such as a two-dimensional allotrope of diamond called graphene (Novoselov et al. 2004). In two dimensions there are precisely seventeen space groups. They are called the 'wallpaper groups', owing to their applicability to the regular patterns on wallpapers.

Defects abound in real crystals, disrupting the infinite translational symmetry present in the mathematical description. There are also systems lying close to the border of periodicity and disorder, such as quasicrystals, which repeat themselves not under translation, as with a crystal, but under a change of scale. This fractal property is a common feature of chaotic systems.

An analogue for everyday life and experienced patterns

On a human scale, repeated activities, like those experienced in everyday life, our daily routines and our habitual behaviours (e.g., the order in which you brush your teeth, or the times of day that you leave and enter your front door) have recognizable repetitions and appear to be predictable (see, e.g., Figure 1). These experienced repetitions follow an understandable pattern, but the pattern is not regular or periodic. Events happen, but perhaps at different times, or in different places or ways. Some actions follow the same steps, but not always in the same order or with the same outcomes.

The existence of pattern and repetitions in our everyday lives, and how we may go about recording and understanding them, is discussed by artists and philosophers. Henri Lefebvre proposes what he terms 'Rhythmanalysis' as a method of interpretation of the orders and repetitions that we experience. He suggests that everyday life is the perceptual interaction of 'natural' rhythms (sleep and waking, mealtimes, etc.) and biological rhythms (heartbeat, for example) with repetitive process linked to homogeneous time (watches and clocks, days and seasons) (Lefebvre 2004: 8–9).

Discovering a theoretical visual analogue for recordings of everyday life and activity are ideas being explored and developed in Lucy's drawings, where she is experimenting with discovering visual equivalents for experienced patterns by describing them as decorative patterns. The basis of this article is an ongoing discussion between the authors regarding the properties of pattern groups, their behaviour in nature, and the applications these may have in designing decorative patterns.

Methods

Decorative repeating patterns feature two basic properties: a motif and a lattice. The motif is the image that is periodically repeated (tessellated), whereas the lattice is the infinite set of points describing the locations of the repetitions. The number of possible lattices is restricted: periodic patterns can feature only two-, three-, four-, or sixfold rotational symmetry in two or three dimensions (Ashcroft and Mermin 1976). Periodic patterns can involve a simple repeat of the motif, translated through the directions and distances dictated by the lattice, or more complicated actions like reflections, rotations and other combinations of translations of the image. Considering all the possible combinations of tessellations and translations of the pattern motif lead to the seventeen wallpaper groups.

The periodic pattern can be described by a unit cell, repeated periodically to reproduce the full pattern. The choice of unit cell has an arbitrariness to it. Any cell which can be tiled in a regular pattern with no gaps so as to reproduce the full image is a legitimate choice. Different choices of unit cell may have different symmetries internally, without this having any bearing on the wallpaper group, which describes the entire infinite pattern.

In our everyday lives we are aware of repetitions, rhythms and routines. We understand them as repetitions in time (like mealtimes) or in space (like sitting on the same chair to eat your breakfast). The individual units of these patterns are ordered in time or in space and it is this ordering, the relationship between repeated units, that defines the pattern, that makes the next unit inevitable. These relationships are the structures of our everyday lives.

This phenomenon is described by Georges Perec:

> A wooden jigsaw puzzle is not a sum of elements to be distinguished from each other and analysed discretely, but a pattern, that is to say a form, a structure: the element's existence does not precede the existence of the whole, it becomes neither before or after it, for the parts do not determine the pattern, but the pattern determines the parts: knowledge of a pattern

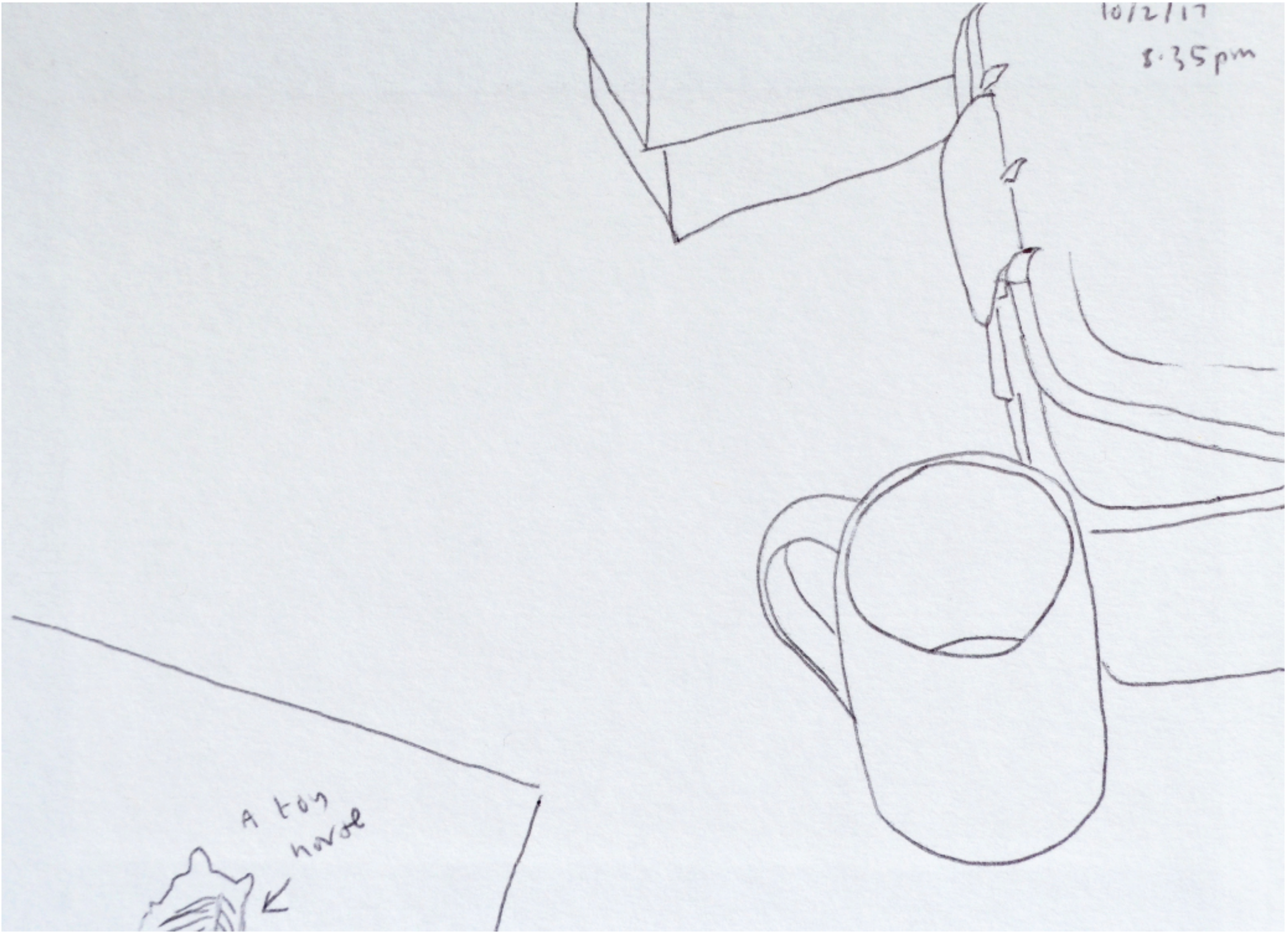

Figure 1: Lucy Ward, Every Cup of Tea I Ever Had *(2017). Pen on paper. 10 February 2017, 8.35pm. © Lucy Ward. Lucy's recordings have taken many forms, and have taken place for a long time: travel journals, diaries and 'collections', recording day-to-day journeys with drawings on maps, records of all her possessions classified into different collections, a record of all the times her newborn baby woke up to feed in the night, a drawing of every cup of tea ever drunk (with a few gaps) since 2007, etc.*

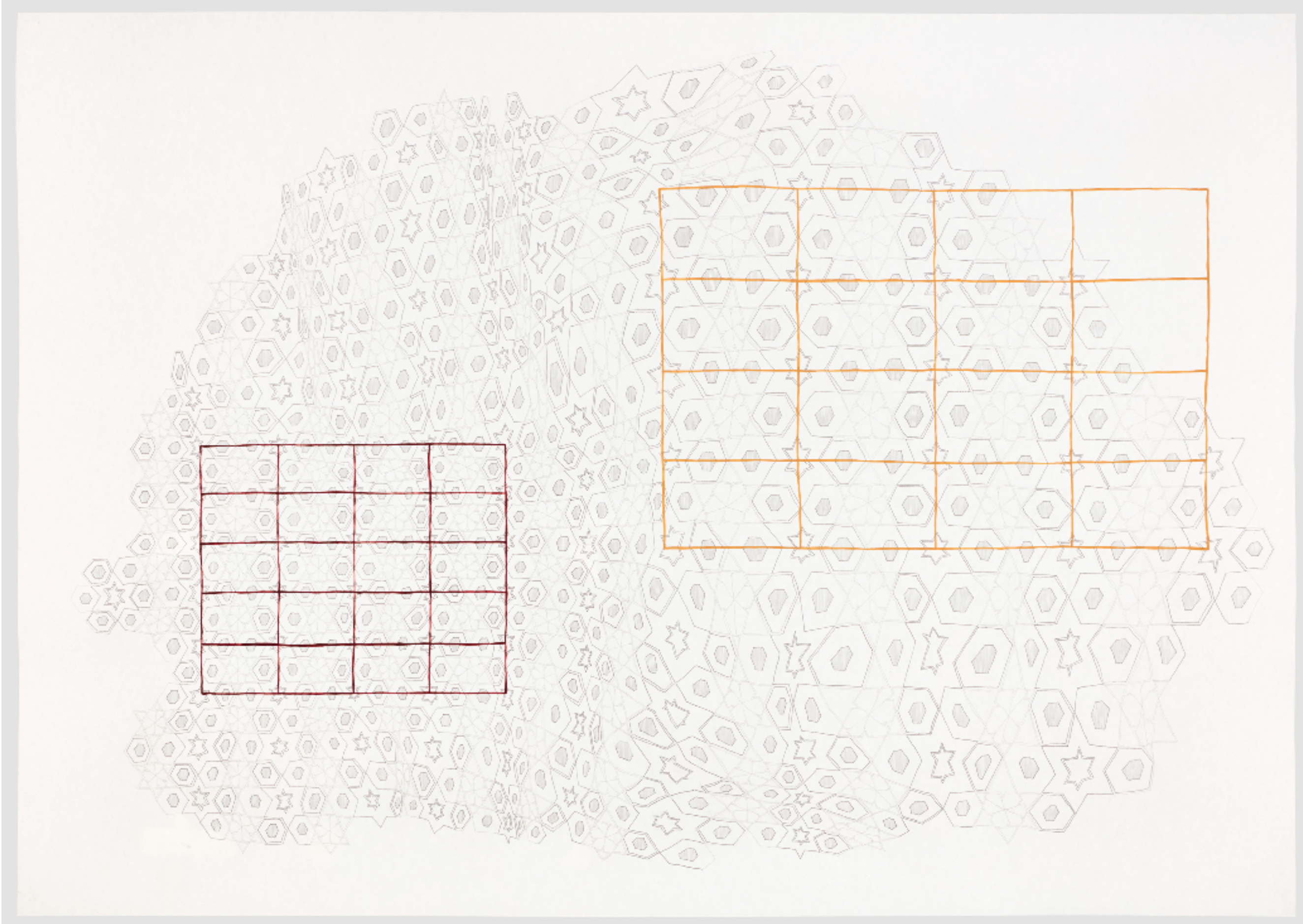

Figure 2: Lucy Ward, Morocco *(2015). Pencil and acrylic on paper. © Lucy Ward. Photograph by Max McClure. Changing the size of parts of the pattern, or the scale of the pattern over the plane were experiments in visualizing a pattern that didn't 'behave'. It was recognizable as the same pattern across the whole image: the repeat was regular in some ways, but not in others.*

> and its laws, of the set and its structure, could not possibly be derived from discrete knowledge of the elements that compose it.
>
> (Perec 1988: Preamble)

This metaphorical overlap suggests that decorative patterns could be a good illustration of the patterns experienced in everyday life: The unit cell of the pattern equating to the event or place, and the pattern system (the lattice) equating to the experience of this pattern. Experienced patterns do not, however, follow such rigid systems. We may understand that the pattern exists, and be able to predict and follow it, but it would not look regular if written down or drawn out. To illustrate this, consider generating a periodic sequence by starting from the letter A, and repeatedly making the substitutions:

A → AB
B → AB

starting from the leftmost letter:

A → AB → ABAB → ABABABAB → ABABABABABABABAB → …

The pattern is periodic and understandable. Now, consider instead making the substitutions

A → AB
B → BA

to the initial letter A, again starting from the leftmost letter:

A → AB → ABBA → ABBABAAB → ABBABAABBAABABBA → …

The sequence appears to be becoming more and more irregular, yet the rules that generate it are just as understandable as in the first instance. The 'Thue-Morse sequence' being described by the second set of substitutions has relevance to many everyday activities: it describes, for example, the sequence in which filter coffee should be poured into two cups to ensure the most balanced strength between cups, or the fairest way for two captains to pick members for their sports teams (Richman 2001). Here, we explore equivalents to this phenomenon of complicated patterns generated by simple rules.

The process of making the drawings also echoes the boundary of regular pattern and disorder explored within the images. Drawing the patterns involves drawing the repeated shapes and motifs of the pattern. This requires a methodical, repetitive formation of the same shapes again and again. The experience of drawing out the patterns is also a repetitive act. The repetitive process of drawing mirrors that of the repetitive routines they describe. And within all this the action of the artist is present within the marks, the 'trace' of their presence. The line does not just describe the pattern, it describes the action of the artist too. Deanna Petherbridge articulates this:

> In the sense that a line is a conduit of meaning or ductus, it induces qualities of movement at the same time as reproducing them: It is both transitive and intransitive.
>
> (Petherbridge 2010: 90)

But the pattern is not a 'true' repeat. The fact it is drawn by hand means that mistakes, inconsistencies and imperfections are inherent. The 'trace' of the artist on the pattern system is to introduce disorder.

Disrupting patterns

The disruption of decorative patterns to find a better equivalent for experienced repetitions could be approached in a number of ways. The disruption could be something small, like drawing-out a pattern (rather than printing or using a computer) and thus the action of the hand varying the repetition. The process of drawing the pattern: the repeated marks, the repetitive gestures following the same set of rules but necessarily different, acts as an index to the repetitions that the drawing represents.

The size of the pattern units, or the pattern scale could also be varied, as in Figures 2 and 3.

Non-regular patterns

The pattern system could also be changed across the page, starting with one kind of symmetry of the pattern motif on one side, and changing it to follow a different set of rules over the course of the drawing and the page.

The drawing shown in Figure 4 poses an idea for ways to transform a pattern from wallpaper group p1 to a pattern in group p2. Group p1 describes patterns with no reflections or rotations. A simple repeat of the pattern motif. Group p2 describes patterns with a twofold rotational symmetry: rotating through half a turn, about the correct points, would return the infinite pattern to itself.

The disorder-within-order in these drawings has important analogues in physical systems. The images in Figures 4 and 5 bear similarities to intermediate states as a system undergoes a phase transition, such as water boiling to steam. Perhaps the easiest-to-visualize analogy would be the development of 'antiferromagnetism'. In a conventional ferromagnet, such as iron, each atom has its own magnetic field (called its spin). These spins sit on a regular periodic lattice and point in the same direction within one magnetic domain (for convenience, label the direction 'up'). We could describe each spin by an arrow pointing from the north pole to the south. Certain systems can undergo

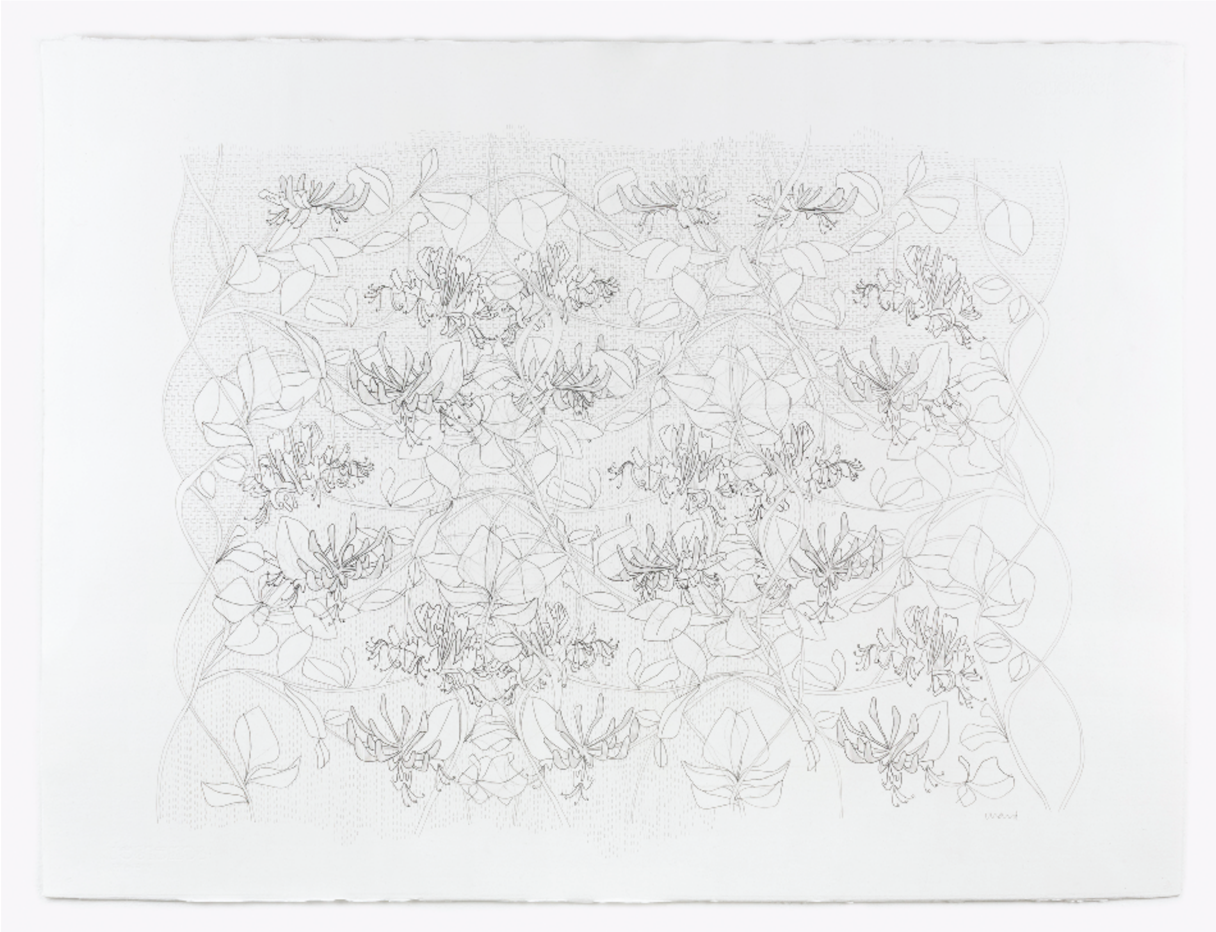

Figure 3: Lucy Ward, Honeysuckle *(2015). Pencil on paper. © Lucy Ward. Photograph by Max McClure. In this drawing the pattern unit is larger than the space it is given in the pattern plane, resulting in the edges of each motif overlapping. The pieces of the pattern are too big to fit into the pattern system, so the pattern is changed and disrupted. The pattern is no longer regular across the page, as the parts of the pattern which are revealed or obscured by the overlap depend on the order in which they are drawn onto the page.*

Figure 4: Lucy Ward, P1 to P2 Fig Drawing *(2017). Pencil on paper. © Lucy Ward. Photograph by Max McClure. This pattern changes from P1 to P2, testing one possible way that this transition may take place.*

Figure 5: Lucy Ward, Study Drawing: Pg to Pm Fig Drawing *(2018). Pencil on paper. © Lucy Ward. Photograph by Max McClure. An example of a drawing changing from Pg to Pm (one glide reflection to one mirror reflection in the x-axis) across the plane of the page.*

instead a phase transition in which antiferromagnetic order develops. In this case half the spins point up, and the other half down, such that each up arrow has only down arrows as neighbours (see Figure 6, Right). Relative to the ferromagnet, this can be looked at as a doubling of the unit cell in every direction: in the left image, an arrow and its surrounding square box can be used as the repeat unit, but in the right picture the minimal unit that can be repeated is a set of four arrows/boxes. This also helps clarify the extent to which the unit cell is arbitrary: the ferromagnet can equally well be described by the antiferromagnetic unit cell, but the converse does not hold.

The left image of Figure 6 has an infinite set of vertical mirror planes, both about the centres of the arrows, and about the midpoint between two. These are the only symmetries other than translations, and the resulting wallpaper group is pm. Moving to the right picture, the symmetries change. There are still vertical mirror planes about the centres of each arrow, but the mirror planes half way between no longer exist. There are, however, new twofold rotation axes about points half way up the wall of each square, as well as glide planes (combined translations and mirrors). The group is called instead cmm.

We tend to just think of the infinite perfect lattice in each case, but during the phase transition there will be a period of time when the new phase nucleates within the old, and the symmetry is broken in one part of the image. This symmetry-breaking process is reflected in Figures 4 and 5.

Quasiperiodic patterns

Attempts to change the size of the motifs, locally, present an interesting challenge. We might expect local size changes to stop the pattern fitting together. One interesting solution is again presented by chaos, in the form of fractals. These patterns return to themselves under a (usually discrete) change of scale, in much the same way that a regular periodic pattern returns to itself under a discrete translation through a vector connecting two lattice points. The name 'fractal' derives from the fact that such structures feature a non-integer, fractional, dimension. As hypothetical as this may sound, fractals often provide excellent approximations to real-world objects, such as clouds, trees, the human circulatory system, boiling water and the universe itself, all of which look self-similar upon zooming in or out to different extents (Strogatz 1994; Gleick 1987).

Figure 7 shows a fractal pattern which lies between periodicity and disorder. It is a Penrose tiling, in this case generated by 'inflation rules' that divide up the tiles in such a way as to return the previous image on a smaller scale (1974: n. pag). The infinite tiling then has a discrete scale invariance. The pattern has a number of fascinating properties. It appears to have points of fivefold rotational symmetry, contrary to the claims at the start of the paper regarding possible rotational symmetries (those results applied only to periodic tilings). While not periodic, the Penrose tiling is not completely disordered either. In fact, it can be generated as a two-dimensional slice through a five-dimensional periodic crystal.

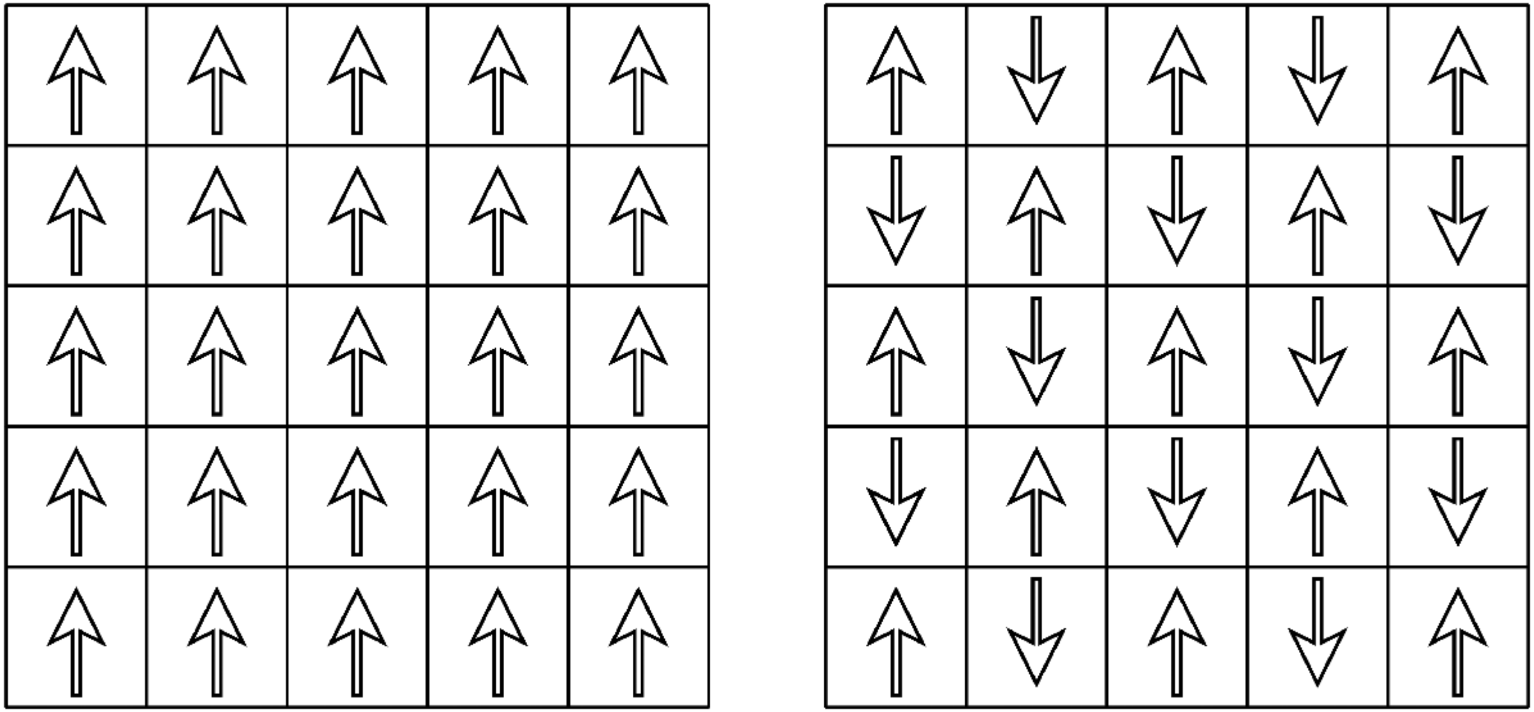

Figure 6: Felix Flicker, The Symmetries of Some Magnetically Ordered States *(2018). © Felix Flicker. Left: the arrows could represent aligned spins describing the magnetic fields of individual atoms within a domain of a ferromagnet such as iron. Right: certain materials can undergo an antiferromagnetic phase transition, in which half the spins point in such a way that nearest-neighbours are always oppositely aligned. The patterns are described by wallpaper groups Pm (left) and Cmm (right).*

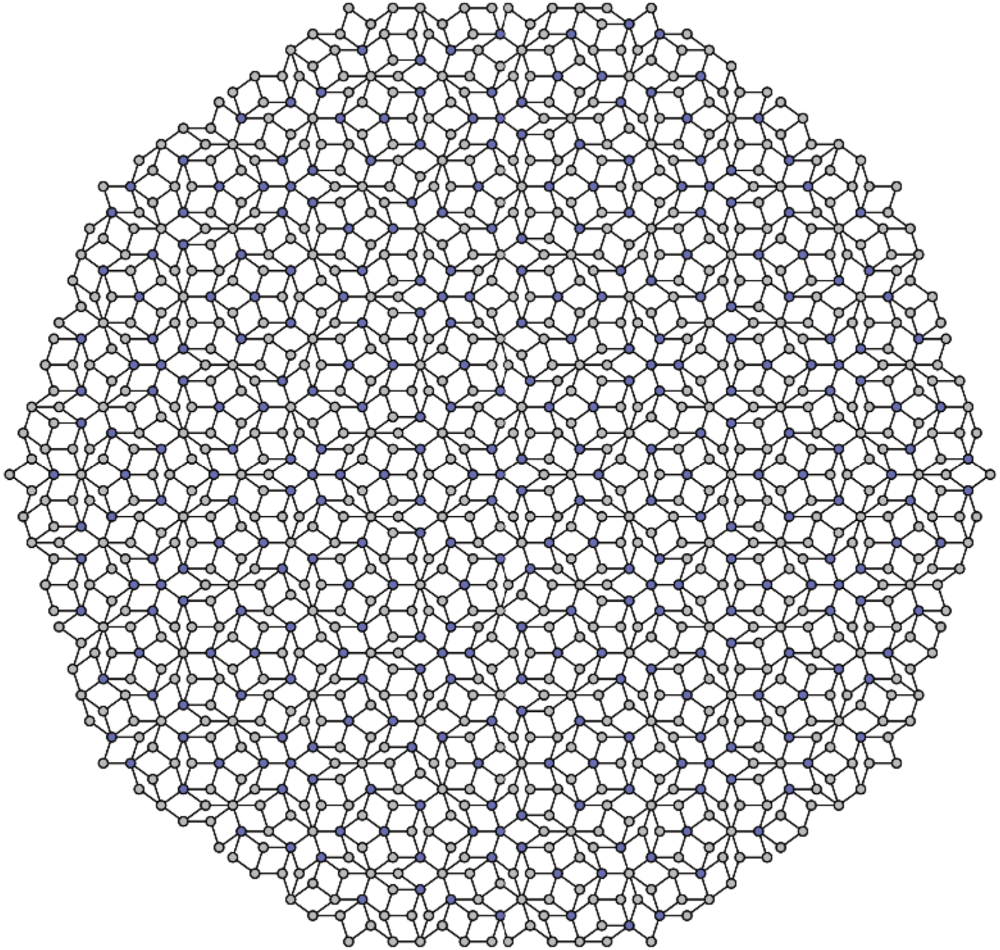

Figure 7: Felix Flicker, A Finite Section of the Penrose Tiling *(2018). © Felix Flicker. A finite section of a Penrose tiling, an example of a quasicrystalline pattern (Penrose74). The image features two different unit cells (a fat and thin rhombus) tiled in an aperiodic manner. The colouring breaks the quasicrystalline symmetry. The image was generated with a computer program written by one of the authors.*

Conclusions

Just as regular repeating events/activities could have visual equivalents in regular visual patterns; it is possible also to find equivalents for more irregular repetitions, the kinds identified in our everyday life. The sense of a pattern in lived experience can be understood in relation to more complex ideas of repeating patterns found in nature, some of which have been referred to here.

Further work will include drawings that experiment with other pattern changes, and more complex ones, like those involving multiple rotations or reflections, or different or changing tessellations.

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